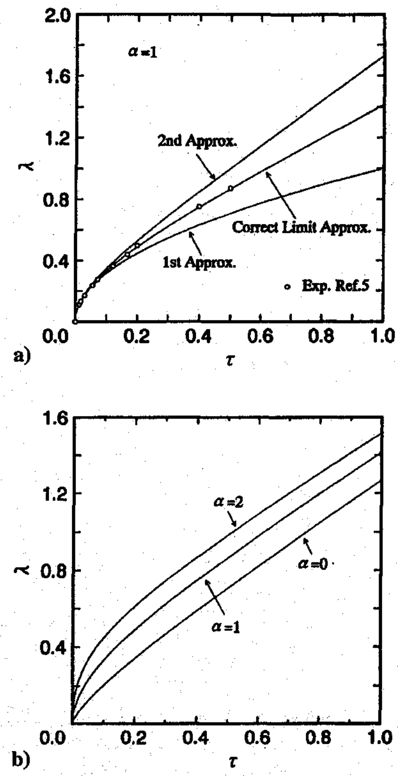

Fig. 1 Shock trajectories: a) comparison of correct limit approximation, experimental data,[5] and Sakurai's approximations for $\alpha = 1$ and $\gamma = 1.4$ and b) correct limit approximations for $\alpha = 0, 1,$ and 2.

As $\lambda \to \infty$, the right-hand side approaches $\lambda^{-3/4}$, in agreement with weak shock decay law. Based on the Rankine–Hugoniot relation and Eq. (22), Jones et al. have obtained another trajectory equation that has been numerically integrated and compared with Eq. (20). As shown by Jones et al., the two trajectories differ by a maximum of less than 8% at $\lambda \sim 1.5$ ($M \sim 1.05$). However, the error decreases rapidly at lesser and greater distances.

Equations (19–21) are valid in the strong and intermediate shock regions and would deviate from the weak shock decay law only in the weak shock region. Therefore, they are still adequate as the first-order correct limit equations and are improvement upon Sakurai's second-order approximations.

Conclusions

A simplified approach has been proposed and applied to the development of cylindrical shock trajectory formula given by Vlases and Jones. Furthermore, the same technique has been used to develop the correct limit equations to predict the shock trajectories for planar and spherical blast waves. For intermediate and weak waves, the results achieved by the proposed method are improvement upon Sakurai's second-order approximations.

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Observations of a Planar Jet Subjected to Large-Amplitude, Low-Frequency Disturbances

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Introduction

THE behavior of jets under the influence of periodic disturbances has been of interest since the mid-1800s, including Plateau's[1] study of gravitationally induced liquid jets issuing from orifices vibrating from acoustical sources in contact with the vessel, and LeConte's[2] observation of the effect of acoustical tones from a musical trio on gas lights while at a dinner party. Brown[3] presented results on the effect of periodic forcing of a three-dimensional jet with Reynolds numbers of 1×10^2 to 2×10^2. The acoustic disturbances were introduced externally at frequencies ranging from 76 to 660 Hz. He observed both increased and suppressed growth of the jet.

A distinct saddle-back shape in the longitudinal velocity profile of a planar jet having nozzle aspect ratios of 20 and 25 and a Reynolds number of 1.3×10^4 was noticed by van der Hegge Zijnen[4] with the maxima near both ends of the profile being, at times, 10% greater than the centerline velocity. The saddle-back profile was accentuated by larger aspect ratios and by closing the ends to maintain two-dimensionality. Sato[5] used a channel approach (7.5–183.3 nozzle widths) to a nozzle with an aspect ratio varying from 10 to 91. He found that the disturbances changed velocity fluctuation levels along the jet centerline and that the Strouhal number equaled 0.23 for the symmetric fluctuations and 0.14 for the antisymmetric fluctuations.

Crow and Champagne[6] found that forced disturbances reduced the potential core of the jet and that the spread and axial velocity decays reached their asymptotic values more quickly than those of a natural jet. Browand and Laufer[7] introduced artificial disturbances and observed large-scale structures and the merging of vortex rings that they labeled pairing interactions. Zaman and Hussain,[8] who studied vortex pairing in air jets subjected to pure-tone acoustic disturbance, found both amplification and suppression of velocity fluctuations depending on the disturbance frequency.

Oster and Wygnanski[9] demonstrated that spreading rates can be significantly increased by disturbing the flow at frequencies at least an order of magnitude below the initial instability frequency. Ho and Huang[10] confirmed the effect of disturbing flows at frequencies lower than the natural instability frequency and noted that the spreading rate was sensitive to low disturbance frequencies and that collective interaction could result in 10 or more vortices coalescing.

The objective of this research is to determine the effect of large-amplitude, low-frequency disturbances on a free planar jet. Low-frequency, subaudible excitation is of interest in applications where higher excitation frequencies may be objectionable. The disturbance frequencies studied extended into the audible range, but the subaudible frequencies (below 20 Hz) are of particular interest.

Apparatus and Procedures

The nozzle, 2.54 cm wide and 119 cm long, was designed to approximate actual diffuser geometries used in building applications.[11] The axial distance was nondimensionalized by the nozzle width and

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Table 1 Reynolds and Strouhal numbers

Frequency, Hz	Re	Re_Θ	Sr	Sr_Θ
0	7272	2.10×10^1	0	0
2	7317	1.73×10^1	0.011	2.66×10^{-5}
10	7241	2.90×10^1	0.057	2.27×10^{-4}
16	7297	2.02×10^1	0.090	2.50×10^{-4}
20	7347	1.66×10^1	0.112	2.52×10^{-4}
30	7350	2.26×10^1	0.168	5.16×10^{-4}
40	7140	2.66×10^1	0.230	8.58×10^{-4}
50	7089	3.83×10^1	0.290	1.57×10^{-3}
56	7109	1.35×10^2	0.324	6.16×10^{-3}

the longitudinal distance by the nozzle length. Periodic pulsing of the jet was achieved by rotating a thin disk in a circular duct located 4.5 m upstream of the nozzle. The natural jet had a Reynolds number, $Re = u_0 B/\nu$, of about 7.2×10^3.

Comparisons of velocity fluctuations between the flow immediately downstream of the disturbance mechanism and at the nozzle showed that the turbulent structures introduced to the flow by the rotating damper were dissipated by the flow conditioning. The fluid structures observed in the jet were not turbulent structures that had been created by and separated from the mechanism used to generate the disturbances and then been carried by the flow to the outlet.

Hot-wire anemometry was used to collect flow information. Both single-wire and two-wire 4-μm tungsten sensors were used. A sampling frequency of 1000 Hz was generally used over a 120-s period.

The disturbance frequency f was nondimensionalized by the nozzle width and the centerline exit velocity (generally about 4 m/s), u_0, to obtain the Strouhal number, $Sr = fB/u_0$. The Reynolds and Strouhal numbers, based on both the nozzle width B and the momentum thickness Θ, are shown in Table 1.

Observations

The transverse profile of the axial velocity for the natural jet achieved a self-similar profile by 10 nozzle widths. The exit transverse velocity profile of axial velocity approximated a top-hat profile. Transverse velocity profiles of the undisturbed jet showed good agreement with those obtained by Heskestad,[12] Gutmark and Wygnanski,[13] and Sfeir.[14]

The axial velocity decay rate and kinematic virtual origin are given by Flora and Goldschmidt[15] as $[U(X,0)/U(0,0)]^{-2} = K(X - C)$, where K is the axial velocity decay rate and C is the kinematic virtual origin. The results are shown in Table 2. The decay rate for the natural case, 0.158, compares closely to those of Gutmark and Wygnanski (0.19), van der Hegge Zijnen (0.205), Sfeir (0.156), Ramaprian and Chandrasekhara[16] (0.168), and Flora and Goldschmidt (0.162 to 0.364). Chambers and Goldschmidt[17] provide a thorough overview of decay rates and jet origins. Disturbing the jet at $Sr = 0.112$ decreased the decay rate by about 4% over the natural case whereas disturbance frequencies about $Sr = 0.112$ increased the decay rate, at times, by over 50%.

The transverse profiles of axial velocity for the natural and disturbed jets collapsed to a universal profile, closely approximated by a Gaussian or hyperbolic secant form when scaled by the jet half-width. The near field of the jet was more strongly affected by Strouhal numbers at or below 0.112 than was the far field. The velocity profiles at five nozzle widths, Fig. 1, showed increased spread with a corresponding decrease in the centerline axial velocity as the Strouhal number increased from 0 to 0.112. However, at 20 nozzle widths downstream of the nozzle, the profiles were similar in shape to those of the natural jet with only a slight increase in the spread of the jet for Strouhal numbers below 0.112. The velocity profiles at 20 nozzle widths became increasingly more distorted and wider than the natural case at Strouhal numbers of 0.168, 0.230, and 0.290. Decreased centerline velocity accompanied the increase in spreading as momentum was transferred from the center of the jet to the mixing layers.

Table 2 Axial velocity decay in the two-dimensional region

Strouhal no.	Approximate start of two-dimensional region, X	Kinematic virtual origin, C	Axial velocity decay constant, K
0.000	7	0.0211	0.158
0.011	7	0.1664	0.158
0.057	7	0.0020	0.157
0.090	8	0.8453	0.156
0.112	10	−1.466	0.151
0.168	9	−0.5996	0.223
0.230	7	0.0193	0.193
0.290	9	−0.9583	0.187
0.324	6	−0.8619	0.201

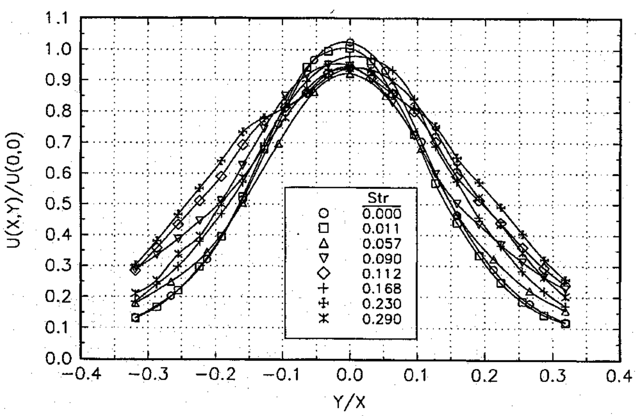

Fig. 1 Transverse profile of axial velocity normalized by exit velocity at various Strouhal numbers, $X = 5$.

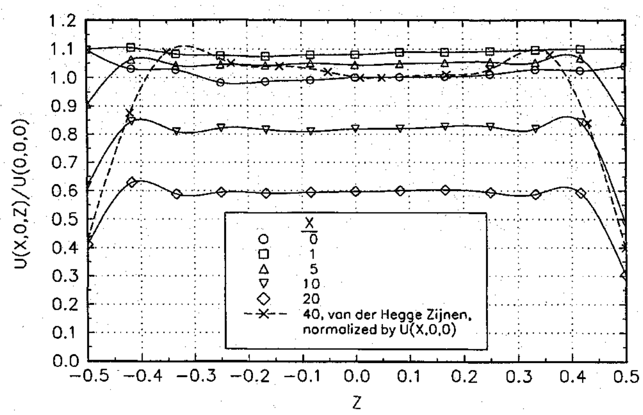

Fig. 2 Longitudinal profile of axial velocity along nozzle centerline at various axial positions, $Sr = 0.000$.

Examination of the longitudinal profile of the axial velocity for the natural jet, Fig. 2, shows results consistent with those of Sfeir[18] and Marsters[19,20] in that the longitudinal width of the natural jet decreased within five nozzle widths from the exit. The jet width increases in the transverse direction and decreases in the longitudinal direction until the transverse width becomes larger than the longitudinal width. This point is referred to as the cross-over point by Krothapalli et al.[21] and is associated with the beginning of the axisymmetric decay region. Figure 2 shows the distinctive saddle-back shape of an undisturbed planar jet, first recorded by van der Hegge Zijnen[22] and later observed by others such as Krothapalli et al.,[23] Sfeir,[24] Marsters,[25,26] and Schwab and Pollard.[27] The velocity at the ends is about 2.3, 3.2, and 5.4% greater than the centerline velocity for 5, 10, and 20 nozzle widths, respectively, from the exit. The peaks grow, relative to the centerline velocity, with axial distance, consistent with the results of Marsters,[28] but not with those of Sfeir,[29] who showed the peaks decreasing from 10 nozzle

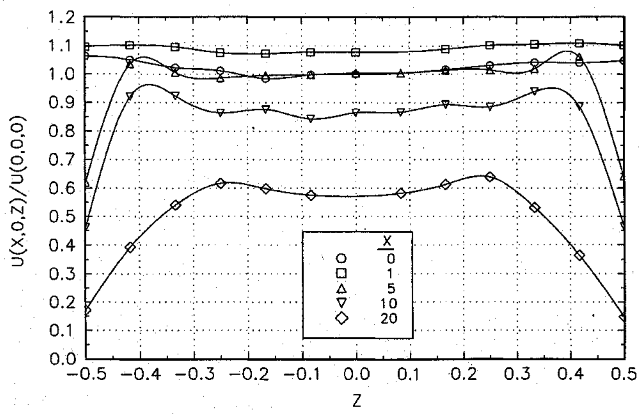

Fig. 3 Longitudinal profile of axial velocity along nozzle centerline at various axial positions, $Sr = 0.112$.

widths to 50 nozzle widths. The results shown for van der Hegge Zijnen were taken at $X = 40$ for a nozzle with an aspect ratio of 40.

Disturbing the jet significantly decreases the longitudinal width of the jet with respect to the undisturbed jet while accentuating the saddle-back shape as shown in Fig. 3 for the jet pulsed at $Sr = 0.112$. The increased transverse width of the pulsed jet shows significant entrainment into the core of the jet that leads to entrainment of the ends of the jet. The increased transverse jet width and the decreased longitudinal width indicate that the forced disturbances cause an early transition from the two-dimensional regime to the axisymmetric regime.

Disturbing the jet results in peak magnitudes of the absolute velocity fluctuations at the jet centerline; in the natural jet the peak fluctuations occur in the mixing layers. Disturbances lead to more uniform velocity fluctuations near the jet exit that are higher than those of the natural jet. Fluctuations in the mixing layer of the disturbed jet are about twice those of the natural jet. Higher velocity fluctuations indicate greater mixing, especially at the center of the jet, and can indicate the passing of large turbulent structures.

High-speed photography revealed that the forcing generated large-scale structures at the nozzle exit. These vortices created large velocity fluctuations. The large-scale structures result in enhanced mixing and reduction of the potential core, leading to a flattening of the velocity fluctuation profile near the jet core as well as more uniform velocity fluctuations relative to the centerline velocity, similar to that observed by Sato.[30]

The half-width of the jet disturbed at $Sr = 0.230$ was nearly 60% greater than that of the natural jet at $X = 5$. At $X = 20$, the spread of the jet disturbed at $Sr = 0.290$ was nearly 40% greater than that of the natural jet. High-speed cinematography and still photography showed that the greatest increase in spread from large-amplitude disturbances is immediately after the exit of the nozzle.

Conclusions

The large-amplitude disturbances of a planar jet increased entrainment in the transverse direction leading to entrainment of the jet ends. Increased entrainment results from the formation of symmetric, large-scale vortical structures and smaller, interactive structures. Reduced longitudinal spreading then accelerated the transition to an axisymmetric form. Accelerated transition to an axisymmetric form further enhances overall jet entrainment, because entrainment increases with X for an axisymmetric jet rather than with $x^{1/2}$ for a two-dimensional jet.

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